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Normalized response distribution expressions for ground-supported rigid rocking bodies

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ABSTRACT

Estimating the seismic response of ground-supported rocking rigid blocks, is a topic that has attracted significant research interest in the past few decades, since it concerns, among others: (a) several modern structures or ancient monolithic columns that utilize rocking as a seismic protection mechanism and (b) numerous free-standing contents (e.g. museum artefacts) located on the ground floor or lower floors of stiff buildings. In the present research work, by means of a parametric study, utilizing two-dimensional rectangular blocks of varying sizes and ordinary earthquake records, the rocking response at increasing intensity levels was assessed through Incremental Dynamic Analyses. Following the demand evaluation and in order to allow for an easier utilization of the findings in practical applications, simplified approximate equations have been obtained via nonlinear regression analysis. The proposed equations provide an estimate of the peak rocking response distribution, expressed in terms of the normalized, to the dimensionless slenderness angle α, peak rocking angle, at increasing ground motion intensity levels.

Keywords: rocking response, overturning, seismic response, free-standing contents

INTRODUCTION

The prediction of the dynamic response of free-standing ground-supported rigid rocking blocks under seismic ground motion excitations has attracted considerable research interest for several decades. The topic concerns a wide spectrum of structural (e.g. columns in cultural heritage structures, chimneys, bridge piers) as well as unanchored or ineffectively anchored acceleration sensitive nonstructural elements (e.g. museum artefacts, computer servers, electric equipment) whose unsatisfactory performance could undermine the community's seismic resilience. Those elements are either explicitly designed to rock within the so called "safe region" (Spanos & Koh, 1984) when they are subjected to a sufficiently strong ground motion that could initiate rocking or unintentionally develop a rocking response since, they fall into the category of free-standing objects in which the rocking motion is not restricted by anchors or any other protection means (e.g. base isolation). The term "safe region" refers to the response domain that is away from the block overturning failure mode.

Owing to the above, there is currently an emerging need for developing simplified response predictive expressions for estimating the rocking response of free-standing rigid blocks, in an analogous manner that simplified expressions exist in the literature for predicting the central value and the dispersion of the nonlinear response in yielding oscillators. The latter are the well-known R-μ-T relationships (e.g. Veletsos & Newmark, 1960; Ruiz-Garcia & Miranda, 2007), where R is the strength ratio, T is the period of vibration, and μ is the ductility demand. The proposed approach utilizes the characteristic frequency p as a substitute to the amplitude dependent rocking period, and consequently offers $I - \tilde{\theta} - p$ expressions, where, $\tilde{\theta}$ is the normalized peak rocking angle and I is a dimensionless intensity measure.

ROCKING THEORY BASIC PRINCIPLES

A rigid rocking block of mass *m*, as the one illustrated in Figure 1, may be considered to have a height equal to 2*h* and a base width equal to 2*b*. Assuming that the friction of the rocking plane is sufficiently high to prevent sliding, the block under a horizontal ground motion excitation $\ddot{u}_g$ will be set to rocking about its pivot points O and O′ when the ground motion acceleration becomes at least equal to $\mathrm{g} \cdot \tan\alpha$, where g is the acceleration of gravity and α is the block stability (or slenderness) angle, that is given by,

$$\alpha = \tan^{-1}\left(\frac{b}{h}\right) \tag{1}$$

The block stability angle α along with the block half diagonal, *R* defined as,

$$R = \sqrt{b^2 + h^2} \tag{2}$$

fully specify the geometry of the rigid block. The equation of motion that provides the angular acceleration $\ddot{\theta}$ during the rocking motion can be expressed as (e.g., Makris & Vassiliou, 2013),

$$\ddot{\theta} = -p^2 \cdot \left[\sin(\alpha \cdot \mathrm{sgn}(\theta) - \theta) + \frac{\ddot{u}_{\mathrm{g}}}{\mathrm{g}} \cdot \cos\left(\alpha \cdot \mathrm{sgn}(\theta) - \theta\right)\right] \tag{3}$$

where, p is the characteristic frequency parameter of the rigid rocking block equal to,

$$p = \sqrt{\frac{3 \cdot \mathrm{g}}{4 \cdot R}} \tag{4}$$

Under pure rocking, when the block returns to its initial position (i.e. when rotation changes sign at $\theta = 0$), impact occurs. One possible way to treat impact is by means of the coefficient of restitution, η that essentially provides a relation between the pre- and post-impact angular velocities, $\dot{\theta}_1$ and $\dot{\theta}_2$, (Housner, 1963) respectively, as

$$\eta = \frac{\dot{\theta}_2}{\dot{\theta}_1} \tag{5}$$

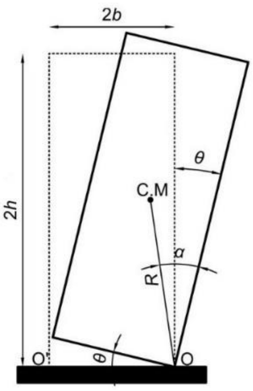

Figure 1. *Rocking block geometry*

METHODOLOGY

For estimating the rocking response for a series of rocking blocks, which will consequently allow us to develop the ready-to-use $I - \tilde{\theta} - p$ distribution expressions, we adopted the standard Incremental Dynamic Analysis (IDA) methodology (Vamvatsikos & Cornell, 2002) with the proposed alterations by Lachanas & Vamvatsikos (2022) for the specific case of rocking structures. For performing the needed time-history analyses on the considered rigid rocking blocks subjected to horizontal acceleration at their base, a suite of 105 ordinary (no long-duration, no pulse-like) ground motion records was selected from the PEER-NGA strong motion database (PEER, 2006). Hence, the only source of uncertainty in estimating the rocking response statistics per rocking block size will be the inherent randomness in the acceleration signature.

To fully define each IDA curve an efficient and sufficient Intensity Measure (IM) along with an Engineering Demand Parameter (EDP) representative of the response of the rocking block are needed. Herein, two dimensionless IMs are adopted, as originally proposed by Dimitrakopoulos & Paraskeva (2015), which will be collectively referred to by the symbol I:

(a) the dimensionless peak ground acceleration, $PGA/(\mathrm{g} \cdot \tan\alpha)$, henceforth denoted as $I_A = PGA/\mathrm{g}\tan\alpha$

(b) the dimensionless peak ground velocity, $pPGV/(\mathrm{g} \cdot \tan\alpha)$, henceforth denoted as $I_V = pPGV/\mathrm{g}\tan\alpha$

With respect to the EDP, the absolute peak rocking angle θ_{max} normalised by the block stability angle α, i.e., $\tilde{\theta} = \theta_{max}/\alpha$, was employed. The first appearance of $\tilde{\theta} > 0$ signifies the onset of rocking, whereas $\tilde{\theta} \geq 1$ can be taken to signify *nominal overturning*. In the process of defining the seismic induced rocking response from the initiation of rocking up to block instability (overturn) the ground motion records were incrementally scaled at increasing intensity levels.

To formulate, to the extent possible, an easy-to-implement probabilistic model for evaluating the response statistics of the rocking response, only those variables that are strongly affecting it will be maintained. With reference to the block size, it was showcased by several past studies (e.g. Housner, 1963; Yim *et al*, 1980) that it is strongly affecting rocking and hence the associated parameter, i.e. the characteristic frequency p, could not be normalized out of the process. Yet, following a rocking response sensitivity analysis (see Figure 2) it was demonstrated that variations in the block stability (or slenderness) angle α are barely noticeable in the computed rocking response for any practical application and hence may be disregarded. This essentially means that a constant value for such a parameter will be assumed for estimating the response statistics in this study (i.e. 0.22 rad) and those will hold for rocking blocks with other slenderness ratios, at least when those are subjected to ordinary ground motion records.

A similar sensitivity analysis was undertaken for checking the effect of the restitution coefficient η on the rocking response. Contrary to the block stability angle α, it was found that even small variations result in notably different rocking responses for the same block size. However, given (a) the dominant and significant dispersion due to the record-to-record variability and (b) the difficulty in determining such a parameter outside of controlled experiments, the restitution coefficient was set to a constant value [i.e. $\eta = 0.92$ as for instance in Giouvanidis & Dimitrakopoulos (2018)]. Consequently, the proposed expressions are anticipated to hold at least for moderate variations of the restitution coefficient around this value.

For the detailed probabilistic assessment on the response of ground-supported rocking blocks, a total of 44 rectangular rigid blocks were considered. Those blocks have frequency parameters p that range from 0.7 s^{-1} to 5.0 s^{-1}, and their dimensions span across the following boundaries:

p=0.7 s^{-1} corresponds to a block with $2b \approx 6.55$ m, $2h \approx 29.31$ m

p=5.0 s^{-1} corresponds to a block with $2b \approx 0.13$ m, $2h \approx 0.57$ m

The response of the rocking blocks was studied utilizing 2D models. Each of the 44 rocking blocks was subjected to one horizontal component of the 105 considered ground motion record pairs. At each intensity level, the rocking response was computed by solving Equation (3) using the software developed by Vassiliou (2021) to estimate the peak rocking angle θ_{max}.

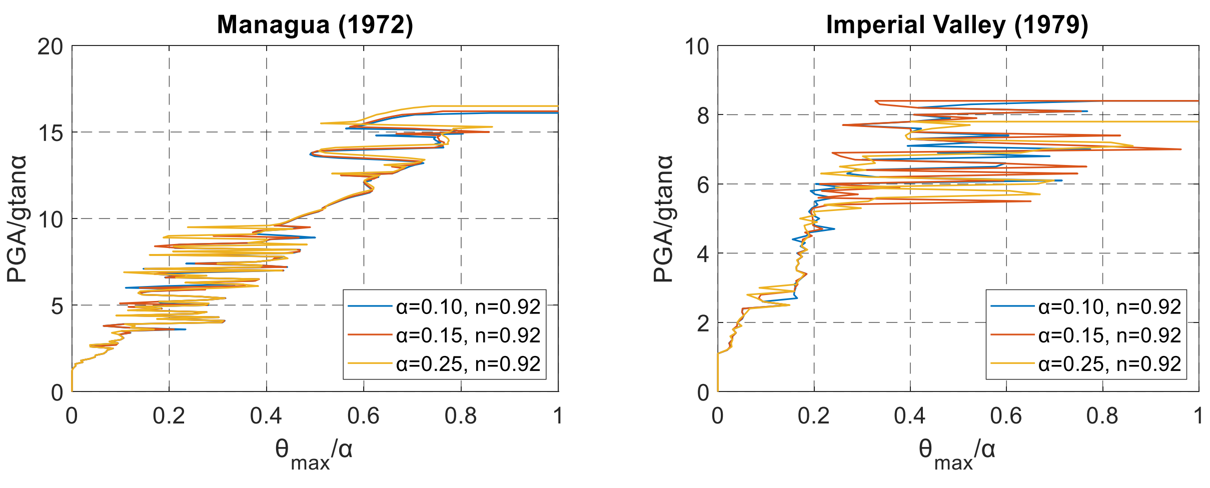

Figure 2. *IDAs for two different earthquake records for variable block stability angles α and a constant coefficient of restitution η considering a rigid block with p=1s^{-1}.*

REGRESSION ANALYSIS OF ROCKING RESPONSES

As discussed earlier, the ultimate scope of this study is to develop simplified, yet sufficiently accurate, rocking response distribution expressions that practicing engineers could easily utilize for computing the seismic rocking response statistics of free-standing rigid blocks that are founded at the ground level. This is deemed to be a rather important aid to seismic design and assessment studies that involve rocking systems. To this end, a nonlinear regression analysis was applied on the computed rocking response data, that are essentially EDP data associated to certain IM levels, to get $I - \tilde{\theta} - p$ expressions for their median and dispersion (i.e. standard deviation of the log of the data). The results are presented here for two *IM*s, these being the dimensionless *PGA* and *PGV* of the horizontal component of ground motion applied to the block.

Median and dispersion fitting for *PGA*

Equation (6) depicts the functional form that is used to relate the median dimensionless EDP, $\tilde{\theta}_{50}$, with the dimensionless *PGA* intensity, I_A, which is the $PGA/g\tan\alpha$. The first part of this expression captures the transition from the very initiation of motion to actual rocking, the second represents the main body of rocking response up to overturning and the third part captures the first occurrence of nominal overturning. The intensity level where overturning occurs is fitted via a separate regression, using an expression that is only a function of p.

$$\tilde{\theta}_{50}(I_A) = \begin{cases} \tilde{\theta}_1 (I_A - C_1)/(1.2 - C_1) & \text{for } C_1 \leq I_A \leq 1.2 \\ 0.1 \cdot A_1 \cdot (I_A - C_1)^{1.25} - \frac{B_1}{100} & \text{for } 1.2 < I_A < I_{A50,ovt} \\ 1 & \text{for } I_A \geq I_{A50,ovt} \end{cases} \tag{6}$$

$\tilde{\theta}_1$ is estimated by the second branch of Equation (6) for $I_A = 1.2$ and $I_{A50,ovt}$ is the median ground motion intensity level that triggers overturning in the rocking block, which can be computed via

$$I_{A50,ovt} = A_2 + \frac{B_2}{p^2} \tag{7}$$

Note that despite using least-squares estimation, we do not employ regression per se to derive the entire distribution in one step. Instead, we are using curve-fitting to derive customized expressions per each statistic of interest, allowing us to casually invert Equation (6) and consequently determine the median intensity, I_{A50}, given $\tilde{\theta}$, which however is not reported here for brevity.

Figure 3 compares the actual median responses against those estimated by means of Equation (6), for a number of indicative p values. Figure 4 illustrates the overturning intensity for the entire range of p considered in this study. Apparently, the proposed rocking response distribution expressions provide a good estimate of the median response for a multitude of blocks and across a wide range of intensities.

Table 1. *Expressions and constants used with Equations (6-7) to define the median values for PGA.*

IM	*A*1	*B*1	*C*1	*A*2	*B*2
$PGA/g\tan\alpha$	$0.4085 \cdot p^{2.6097}$	$0.4514 \cdot p^{2.7299}$	1.0000	1.1142	8.8431

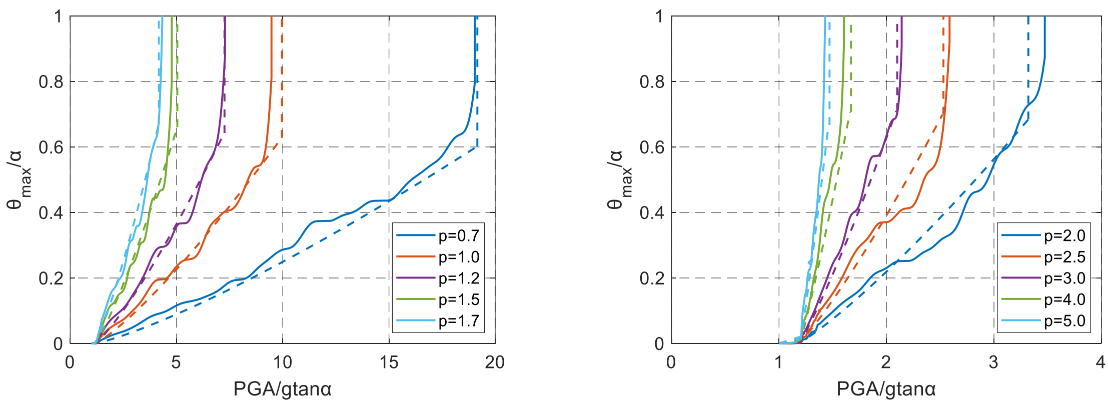

Figure 3*. Median seismic demand estimates for rocking blocks with different characteristic parameters over a range of intensity levels. Results are presented for the dimensionless PGA. Fit is shown with dashed lines.*

A full probabilistic model cannot be considered complete without offering dispersion estimates. In this study, the dispersion estimates at each intensity level were computed as the half-distance between the logarithm of the 16% and the 84% percentiles typically assuming a lognormal distribution. Similarly to the fitting approach adopted for the median, the curve fitting process on the dispersion curves of Figure 5 yielded for the dispersion the following expression,

$$\beta_A(\tilde{\theta}) = \begin{cases} A_1 \cdot \frac{\tilde{\theta}^{B_1}}{e^{\tilde{\theta}}} & \text{for } 0 \leq \tilde{\theta} \leq 0.8 \\ \beta_A(\tilde{\theta} = 0.8) & \text{elsewhere} \end{cases} \quad (8)$$

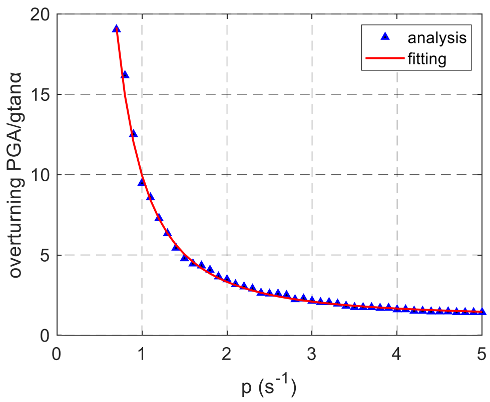

Figure 4. *Median seismic demand for the first occurrence of overturning considering the dimensionless PGA as an IM.*

Table 2. *Expressions used with Equation (8) to define the dispersion for PGA.*

IM	***A₁***	***B₁***
$PGA/\text{gtan}\alpha$	$0.0420 \cdot p^3 - 0.3719 \cdot p^2 + 0.6205 \cdot p + 1.6220$	$0.0088 \cdot p^3 - 0.1302 \cdot p^2 + 0.5635 \cdot p + 0.0581$

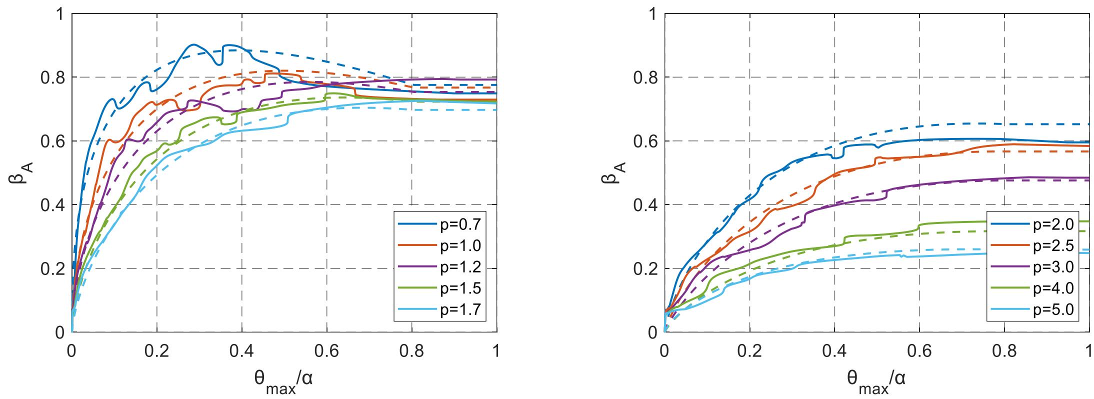

Figure 5*. Dispersion estimates for rocking blocks with different characteristic parameters. Results are presented for the dimensionless PGA. Fit is shown with dashed lines.*

Median and dispersion fitting for *PGV*

An identical fitting approach to the one adopted for the rocking response data obtained considering the *PGA* as an IM was also utilized for *PGV*. Equation (9) depicts the functional form that is used to relate the median dimensionless EDP, $\tilde{\theta}_{50}$, with the dimensionless *PGV* intensity, I_V, or $PGV/\text{gtan}\alpha$.

$$\tilde{\theta}_{50}(I_V) = \begin{cases} 0.001(I_V - I_{V1})/\,(I_{V2} - I_{V1}) & \text{for } I_{V1} \leq I_V \leq I_{V2} \\ A_1 \cdot (I_V - I_{V1})^{1.5} - \frac{B_1}{1000} & \text{for } I_{V2} < I_V < I_{V50,ovt} \\ 1 & \text{for } I_V \geq I_{V50,ovt} \end{cases} \quad (9)$$

where,

$$I_{V1} = C_1 \cdot p \tag{10}$$

I_{V2} is estimated by the second branch of Equation (9) for $\tilde{\theta} = 0.001$, and $I_{V50,ovt}$ is the median ground motion intensity level that triggers overturning in the rocking block (see Figure 7), which may be computed by

$$I_{V50,ovt} = A_2 \cdot p^4 + B_2 \cdot p^3 + C_2 \cdot p^2 + D_2 \cdot p + E_2 \tag{11}$$

Note that Equation (9) could be also inverted to determine the median intensity, I_{V50}, given $\tilde{\theta}$. As can be inferred by inspecting Figure 6, the proposed simplified expressions provide a good approximation of the median rocking response across a range of intensities and for a spectrum of block sizes.

Table 3. *Expressions and constants used with Equations (9-11) to define the median values for PGV.*

IM	*A*₁			*B*₁		
$pPGV/\mathrm{gtan}\alpha$	$0.0468 \cdot p^3 - 0.3018 \cdot p^2 + 1.7193 \cdot p - 0.3845$			$-0.1743 \cdot p^3 + 3.2451 \cdot p^2 + 1.4941 \cdot p - 2.4536$		
IM	C_1	A_2	B_2	C_2	D_2	E_2
$pPGV/\mathrm{gtan}\alpha$	0.0919	0.0147	-0.1899	0.8917	-1.7937	1.9373

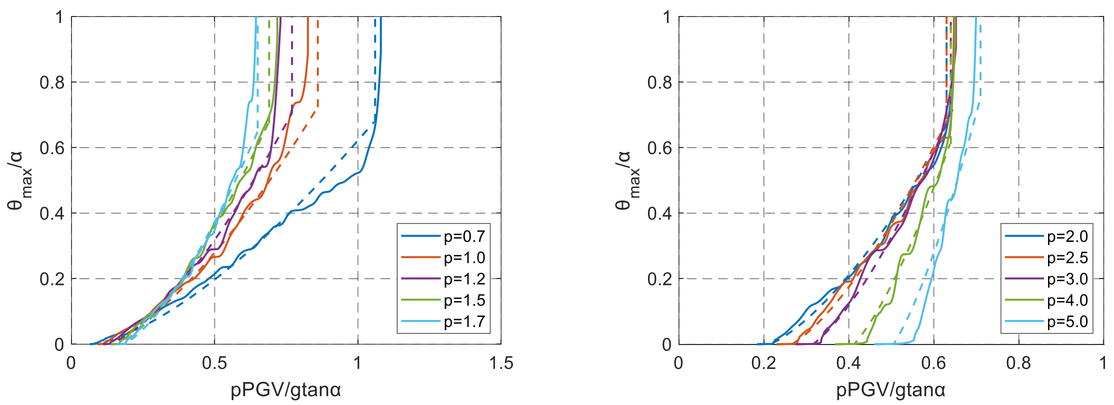

Figure 6*. Median seismic demand estimates for rocking blocks with different characteristic parameters over a range of intensity levels. Results are presented for the dimensionless PGV. Fit is shown with dashed lines.*

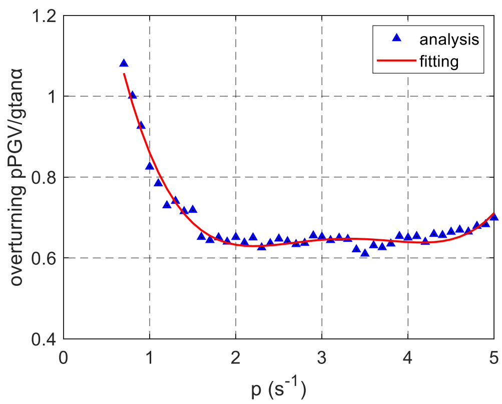

Figure 7. *Median seismic demand for the first occurrence of overturning considering the dimensionless PGV as an IM.*

A two-part expression is also formed by undertaking a nonlinear regression analysis for the dispersion of I_V,

$$\beta_V(\tilde{\theta}) = \begin{cases} D_1 - A_1 \cdot \frac{\tilde{\theta}}{(\tilde{\theta}+B_1)^{C_1}} & \text{for } 0 \le \tilde{\theta} \le 0.7 \\ \beta_V\left(\tilde{\theta} = 0.7\right) & \text{elsewhere} \end{cases} \tag{12}$$

The fit illustrated in Figure 8 with the dashed line is considered satisfactory and slightly on the conservative side in regions of high deviations. Hence proposing more complicated expressions was deemed to be an unnecessary refinement.

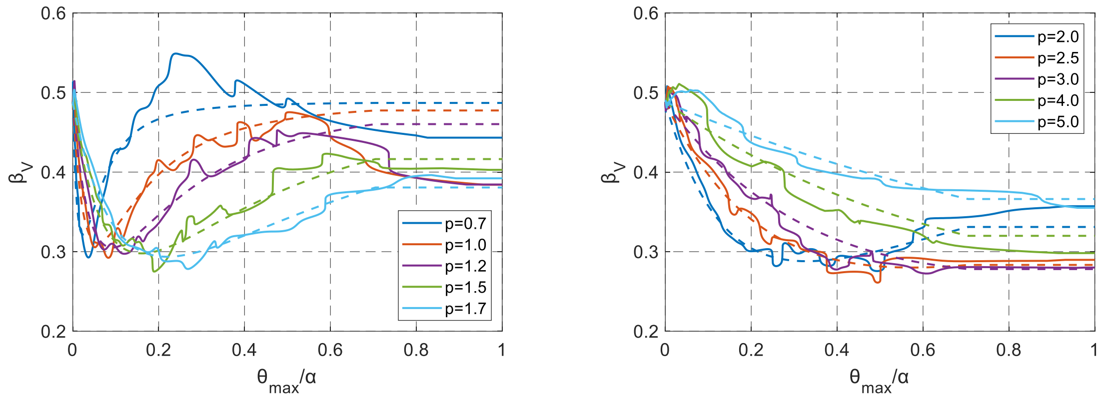

***Figure 8**. Dispersion estimates for rocking blocks with different characteristic parameters. Results are presented for the dimensionless PGV. Fit is shown with dashed lines.*

***Table 4.** Expressions used with Equation (12) to define the dispersion for PGV.*

IM	A_1	B_1	C_1	D_1
$pPGV/\mathrm{g}\tan\alpha$	$0.0090 \cdot p^{7.6659}$	$0.1750 \cdot p^{2.4969}$	4	0.4880

CONCLUSIONS

Given the potential of the rocking motion to constitute a favorable seismic response mechanism for a variety of structural configurations and its unintentional occurrence in unanchored nonstructural elements, the analytical modeling of rocking blocks has attracted considerable research interest. The accurate prediction of the seismic induced rocking demands under a particular ground motion record remains a problem of substantial complexity due to the spectrum of uncertainties that are likely to affect the outcome. Yet, owning to a well-established observation that the acceleration signature is the dominant uncertainty source which masks the remaining modeling uncertainty sources, the problem was studied here from a probabilistic standpoint. Hence, the presented study utilized 105 ordinary ground motion records to capture the record-to-record variability and consequently numerically assess the distribution of the peak rocking responses in free-standing rigid rocking blocks of variable sizes. Then these data were exploited by means of a nonlinear least-square regression analysis to develop a set of simplified analytical expressions for estimating the median and the dispersion of the peak rocking response as a function of the characteristic frequency of the block and the intensity of the ground motion. The proposed expressions cover the entire range of the rocking responses, from the initiation of rocking to overturning, and can be readily used in design and performance-based assessment applications. A more comprehensive set of equations with additional details on derivation and applicability can be found in Kazantzi *et al*. (2021).

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