A Dürüm Döner View of Seismic Risk Assessment

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Abstract
A dürüm döner (DD) is a magnificent culinary invention, without which any visit to Turkey would never be complete. Its excellent combination of bread and meat, with the occasional mix of tomato, onions and mayonnaise, is readily available to complement any walk around the sokaks and plazas of any Turkish city and town. The experience is inherently addictive, and Dr. V is no stranger to it. Contemplating a sabbatical in Turkey comes with an increased hazard of ample DD availability, and heightened risk of high calorie intake. When a powerful stakeholder (Mrs. V) steps in to question this sabbatical plan, how is Dr. V to show that he will be able to conduct research while safeguarding his enviable cuddly figure? Three nearby vendors of excellent but highly variable DDs, one exposed yet enterprising researcher, and one tough uncompromising stakeholder come together in a nail-biting risk assessment drama to play out in view of the Bosporus.

Keywords: risk, earthquake, dürüm döner, interface variable.

Dramatis Personae

Dürüm Döner (DD): A hearty mix of sliced doner meat, tomato, onions and mayonnaise, wrapped in a soft durum flatbread. Easily available in many corner shops, each DD represents a potentially hazardous high calorie *event*, typically ranging within 600 – 1300 calories, that cannot be avoided due to its deliciousness. One simply has to live with it and mitigate the consequences.

DD vendor: A shop selling DDs, three of which are in proximity to the hotel that Dr. V likes to frequent. Excellent quality, fast service, but sometimes inconsistent proportions of meat versus tomato, onions, mayonnaise, and bread, can make for an explosive, tasteful, yet highly variable mix of calories. Each shop is a *hazard source*.

Dr. V: A researcher of risk, self-styled connoisseur of fine cuisine and a DD afficionado of monumental proportions. He is internationally known for his well-proportioned (i.e., rounded) figure, and love of good food. He is contemplating a sabbatical in a Turkish University, but he is wary of the risk posed by the easy availability of DDs, especially in light of the take-no-prisoners stance of Mrs. V. Let's just call him the *exposure* or *asset-at-risk*.

Mrs. V: The wife of Dr. V, a strong woman of powerful opinions and an unrelenting interest in keeping Dr. V healthy and good looking. She is the ultimate *stakeholder*.

PhD student: She is a graduate student, working with Dr. V towards her PhD. Her job is to quantify the risk and ensure that it is acceptable. For the case at hand, she will only report to Mrs. V and she wants to do the best job possible to make sure that there is no bias in the assessed risk. To do so, she needs to make sure that the best available evidence is employed, while ensuring that the assessment is done in a timely fashion and with a reasonable budget. Despite working with Dr. V, she knows the consequences of crossing Mrs. V, and she is invested in doing things right to protect her advisor from DD hazard. She is well-versed in probabilistic seismic hazard assessment and seismic risk quantification, but to tackle this new challenge she will need to go back to the basics. Let us call her the *risk analyst*.

The Mission

Dr. V has stated that he would like to have an average of one DD per day, or equivalently, a mean daily rate of $1d^{-1}$ or a mean annual rate of $365yr^{-1}$. This does not mean that every day he will have exactly one DD. Some days he will have one, other days he will have two or more. It also does not even mean that if he does not eat one DD today, he will have two DDs tomorrow, nor does it mean that at the end of the sabbatical year he will have eaten exactly 365 DDs (assuming a non-leap year). In other words, DD events are independent and the process is memoryless, characterized by a constant rate that manifests itself as the overall annual average over many potential realizations of the sabbatical year. For probability enthusiasts, DD events follow a Poisson process.

The site of interest is a hotel overlooking the Bosporus. Within easy distance lie three high-quality DD vendors that Dr. V equally loves and will typically only buy from them. We can assume uniform, equal preference for each vendor.

In order for Dr. V to maintain his calisthenic figure, he has to stay within strict caloric limits. Mrs. V has demanded that the mean daily rate of DD-related calories does not exceed 1000 kcals. Again, this does not mean that Dr. V cannot have a DD with more than 1000 kcals. After all he does not carry with him the equipment to make such an assessment. Some DDs will be higher than 1000 kcals, others will be lower. It is important that on average, the mean daily rate remains lower than 1000 kcals/day.

Our troubled risk analyst needs to quantify the mean daily rate of DD calories and offer the evidence that Mrs. V requires (a process called *decision-support*) to authorize or cancel the sabbatical. It is of utmost importance that she provides unbiased data to support a fair decision. A conservative assessment (biased high) could curry favor with Mrs. V, but seriously anger Dr. V, while an unconservative estimate would bring out the wrath of Mrs. V. In both cases, her PhD is in jeopardy, and she needs to thread the needle, walk the line, do her utmost best to ensure an objective decision. To achieve this, she will need to dive deep into the core of risk assessment and reevaluate her choices in tackling hazard and risk.

The Risk Assessment Basis

The basis for assessing the probability or mean annual frequency of occurrence of complex event A is the total probability theorem:

$$P(event\ A) = \sum_i P(A \mid E_i) \cdot P(E_i) \tag{1}$$

Where E_i are mutually-exclusive collectively exhaustive "sub-events". Here, A can be described as "DD exceeds 1000kcals" per the stated mission. We can further break down Eq. 1 using another layer of subevents, E_{ij}, per each E_i:

$$P(event\ A) = \sum_i P(A \mid E_i) \cdot \underbrace{\left[\sum_j P(E_i \mid E_{ij}) \cdot P(E_{ij}) \right]}_{P(E_i)} \tag{2}$$

Clearly, we can keep chopping up event A to our heart's desire and keep chaining this conditioning from one layer to another until we reach A. Alternatively, we can also forego all this complex chaining of simpler to more complex events, and just go directly to the lowest, most populated layer:

$$P(event\ A) = \sum_{i,j} P(A \mid E_{ij}) \cdot P(E_{ij}) \tag{3}$$

The difference between Eq. 1 and Eq. 3 is clear. Eq. 3 advantageous over Eq. 1 as long as the partitioning offered by *Eij* is superior to the one offered by E_i in terms of the individual probabilities being easier to quantify than those in Eq. 1. In structural engineering terms, these two equations are the equivalent of the finite element method, whereby a structure is cut-up into pieces (the finite elements), for each of which we can easily assess forces and deformations. Whether to use a high- or low-detail mesh is precisely the same question as using Eq. 3 versus Eq. 1.

In that case, what is the role and usage of Eq. 2? Is there any advantage to be gained by employing the clearly more complex process of Eq. 2 against the resolution-equivalent and conceptually simpler Eq. 3? In general, the question may be rephrased as why to prefer the *conditional* approach (Eq. 2) versus the *non-conditional* one (Eq. 3) (see Bazzurro et al. 1998). Our risk analyst needs to re-investigate the risk assessment basis of the dürüm döner problem, and carefully select the approach to employ in her quest to help Dr. V.

The Non-Conditional Approach

Our trusty risk analyst proposes the following process: Say that over 100 days, she samples one DD from each of the three vendors favored by Dr. V. Each of the total 3×100 = 300 DDs is taken to the lab and a calorimetric test is performed to estimate the actual kcal content of the DDs. The probability mass function (or the histogram) of the DDs is drawn and from the corresponding cumulative distribution function, *P*(DD exceeds 1000 kcals) can be estimated by anyone who has sat in the first few lectures of an undergraduate Probability & Statistics course (Figure 1).

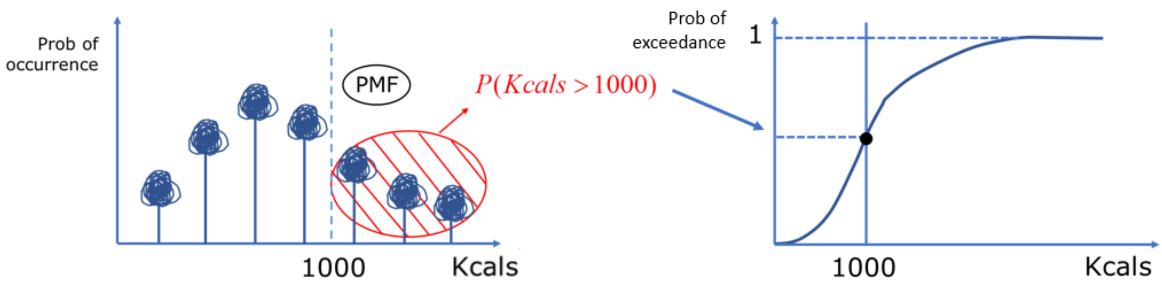

Figure 1. Probability mass function (left) and cumulative distribution function (right) of DD calories, indicating the limit-state threshold of interest.

Conceptually, the method is attractive by virtue of its simplicity. Practically, though, it poses some difficulties:

- One has to wait for 100 days to get adequate events, testing the patience of Dr. V and Mrs. V.
- Calorimetric tests are not cheap. Running 300 of them, even at a bulk price can easily exceed the budget of our risk analyst.

As expected, theoretical simplicity does not necessarily imply actual practicality, and the DD problem is not immune to this phenomenon.

The Conditional Approach

Going the conditional way requires defining an intermediate partitioning of E_i, or a proxy variable to represent the calories in each DD. Hopefully, this will be easier to estimate. This is the so-called *interface variable*. Selecting a proper one is of paramount importance for applying the conditional approach, and its success or failure in many ways hinges on its properties. For example, one horrible option would be to employ the x/y/z coordinates and essentially use a knife to chop the DD into smaller pieces according to said coordinates. It is clear that this will only generate smaller DD pieces that will probably be no less difficult to assess regarding their calorie content. One bad choice has made our problem worse.

An improved option would be to separate the DD into its five basic ingredients of meat, tomato, onions, mayonnaise and bread, and measure their individual weights. In essence, our interface variable will be a five-dimensional vector that fully describes each DD at hand. Then, one could even employ pre-determined values for the calorie content per gram of each ingredient (with the proper distribution due to uncertainty) and get a fairly accurate estimate of the DD calories. It seems like a perfect solution, but is it practicable? Separating the ingredients of a DD after it has been wrapped is not exactly easy. Our risk analyst would have to plead with the owner to let her weigh each of the five before they are wrapped together, clearly risking being thrown out of the shop of a justifiably enraged gentleman. In other words, a vector interface variable offers excellent resolution, at the cost of complexity.

Let us look at some scalar options. The length of the DD may be one consideration, but unfortunately not every owner uses the same flatbread dimensions. Plus, how is one to know if a longer DD does not contain more tomato, while a shorter one may contain more meat (and calories)? How is one to know if the diameter remains the same in the longer and the shorter DD? There is a good chance that such differences depend on the vendor, and how heavy handed he is with respect to the meat and mayonnaise, thus raising the prospect of site-dependence and associated bias. The diameter of the DD (on its own) is plagued by the same issues, but how about the weight? It is still not perfect as it cannot distinguish between meat versus onion weight, but it is a clearer indication of the size of the DD, much better than just the length or just the diameter on their own. Still, how to ensure that site-dependence is not an issue? If the analyst wants to make sure that her DD-weight-to-kcal transformation is unbiased, she needs to be consistent in her DD sampling with what nature (or in this case Dr. V) would do. Taking all her DD samples from the same vendor would not be a wise strategy. Instead, she should perform *hazard-consistent selection* of DDs (see for example Baker and Cornell 2006, Bradley 2010), sampling uniformly from each of the three vendors, following the equal preference displayed by the force of nature called Dr. V.

Finally, we are ready to form an approach that can lead us to a good solution within a reasonable budget, spending only one or two days of our analyst's time. It would involve taking a sample of, say 4 DDs from each vendor, preferably spaced over the duration of one or two days to capture differences among changing employees/cooks, and testing the total or 3×4 = 12 samples by a simple weighing process, or a more expensive lab test to determine the calories. There are many options on how to set up such lab tests, see for example Jalayer and Cornell 2009, or Vamvatsikos and Cornell 2002. The set of obtained results can then be employed via Eq. 3 to assess the year-long risk of Dr. V.

End Game

In the end, the grand question remains: Are we sure that our approximation is good enough to solve the problem with reasonable accuracy? As in most questions in risk assessment, the answer is that it depends on the details. At the very least, we can say that the simplified approach offered above can lead to a good unbiased answer as long as some care is exercised when selecting an interface variable and sampling DDs. Simply put, if you are careful, the conditional approach can offer a budget-conscious solution that is superior to the non-conditional one.

Most importantly, though, if you know how to solve the DD problem, you have the blueprint for solving any risk assessment problem, be it earthquake, wind, tsunami or otherwise. In our case, one only needs to do the following substitutions to fully map DDs to earthquakes:

- Dr. V → building/infrastructure/portfolio/assets at risk
- Kcals → damage/loss
- DD → seismic event / ground motion
- Weight → intensity measure

- DD selection → ground motion selection
- DD vendors → seismic faults
- DD Kcal determination → nonlinear response history analysis

Of course, our solution is not the final word in this problem. As time goes by, computers will grow more powerful, ground motion catalogues will become more populated, and physics-based ground motion simulation will become easier. Then, the favored approach to our seismic risk assessment problem may easily shift. Still, one advice I can give is that if something makes sense for the dürüm döner in the future, it will probably also be valid for earthquakes. So, make your choices wisely.

Acknowledgements

A great deal of thanks goes to Dr. P. Bazzurro, who gave me the opportunity to teach risk assessment in the spring of 2014, starting me on a path of BLTs, paninos, pita gyros, and ultimately dürüm döners. Financial support has been provided by the European Commission through the Horizon 2020 programs “PANOPTIS-development of a decision support system for increasing the resilience of transportation infrastructure based on combined use of terrestrial and airborne sensors and advanced modelling tools”, Grant Agreement number 769129, and “HYPERION–Development of a decision support system for improved resilience & sustainable reconstruction of historic areas to cope with climate change & extreme events based on novel sensors and modelling tools”, Grant Agreement number 821054.

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