Updating structural FE models of cultural heritage assets based on probabilistic tools

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The deterioration of Cultural Heritage assets due to the climatic change and natural hazards is a pressing issue in many countries. In this sense, the assessment of their actual structural integrity based on higher-scale structural responses is key to assess the resilience of these important assets. This paper proposes a rational methodology to integrate modal vibration data into structural FE models based on probabilistic tools. The methodology is based on solid Bayesian probabilistic principles thus allowing uncertainty quantification in the assessment. A real case study for a sixteenth century heritage building in Granada (Spain) is presented. The results show the efficiency of the proposed methodology in identifying the probability density functions of basic material parameters such as the Bulk modulus of the building stones or the modulus of soil reaction among others.

Keywords: Ambient vibration test, Bayesian system identification, cultural heritage buildings, finite element models, global sensitivity analysis, operational modal analysis.

1. Introduction

There are evidences of accelerated deterioration of Cultural Heritage (CH) assets as a consequence of natural hazards. The assessment of the structural integrity of these important assets through monitoring technologies is key to ensure their resilience to natural hazards such as climate change. The development of high-resolution structural models as updatable *digital twins* of the CH assets is a promising solution for a rational and informed decision-making. However, these higher-scale structural models involve a large number of uncertain parameters that need to be calibrated with experimental campaigns. Moreover, these parameters should take into account different sources of uncertainty such as measurement error and model errors and lack of knowledge. This assessment can be efficiently carried out through the Bayesian analysis (Rus et al., 2016). A number of researchers have investigated the problem of model parameter estimation in structural model updating using a Bayesian framework (Beck & Au, 2002; Cheung & Beck, 2009; Lam et al., 2015, Chiachío et al., 2015). However, in some cases, the Bayesian updating involves heavy computation and computational techniques such as parallel computing or surrogate modelling (Hadjidoukas et al., 2015; Chiara et al., 2020) would be required to reduce the computational burden.

In this paper, a rational methodology for FE model parameter updating of cultural heritage assets based on the Bayesian inverse problem is presented. To this end, experimental data about natural frequencies is used from Ambient Vibration Tests (AVT) and obtained using Operational Modal Analysis (OMA) algorithms. A complete building model is developed using a reduced Finite Element model (FEM) to substantially decrease the

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computational burden, and the selection of the most influential model parameters in the system response is carried out through a Global Sensitivity Analysis (GSA) (Saltelli et at., 2008). The selected model parameters are updated through the Bayesian inverse problem using the first the two experimental natural frequencies as system output. As an illustrative example, a real case study for a sixteenth century heritage building in Granada (Spain) is presented. The results show the efficiency of the proposed methodology in identifying the probability density functions of the basic mechanical parameters representing the FE model, such as the bulk modulus or the soil stiffness.

2. Methodology

2.1. *CH asset*

In this research the San Jerónimo Monastery (Figure 1), a sixteenth century CH building in Granada (Spain), is analysed. The CH building has a floor dimension of 57x24 metres and a height of 30 metres in the central nave, 35 metres in the dome and 46 metres in the bell tower.

Figure 1. Exterior view of San Jerónimo Monastery

The building was continuously monitored using accelerometer sensors for the identification of its modal parameters (natural frequencies and mode shapes) through Ambient Vibration Testing (AVT) and Operational Modal Analysis (OMA). In addition, a reduced order Finite Element Model (FEM) was developed using construction drawings dated 1963-69. This model is parameterised taking the bulk modulus of the stone and the soil reaction modulus as main model parameters for this study.

2.2. *Parameter selection by Global Sensitivity Analysis (GSA)*

GSA aims at identifying the most influential model parameters among the set of uncertain parameters. In this research the GSA is carried out based on the first-order index S_i, an index describing the global response variance due to uncertainties in the input model parameters. The computation of S_i is based on the following expression (Saltelli et at., 2008; Sudret, 2008):

$$S_i = \frac{V_{x_i}\left(E_{x_{-i}}(y|x_i)\right)}{V(y)} \quad (1)$$

where x_{-i} denotes the input model parameters other than x_i. The numerator can be evaluated using double-loop Monte Carlo (MC) sampling. In the inner loop, the conditional mean $E_{x_{-i}}(y|x_i)$ is calculated by evaluating the model considering n_1 random variations in x_{-i}. Next, the outer loop computes the variance $V_{x_i}\left(E_{x_{-i}}(y|x_i)\right)$ considering n_2 random variations in x_i. The denominator is the unconditional variance of the model response $V(y)$ and it is evaluated using n_3 random variations of the complete set of input model parameters.

2.3. *Bayesian model parameters identification*

Within the Bayesian approach, the interest is in obtaining the probability density function (PDF) of the input model parameters θ={E, k} given the experimental data D={f_1, f_2} for a specific candidate model class M that idealises the CH building. In this sense, the Bayes' theorem is be used to obtain the required PDF as follows:

$$p(\theta|D,M) = \frac{p(D|\theta,M)p(\theta|M)}{p(D|M)} \quad (2)$$

where $p(\theta|D,M)$ is known as the posterior distribution of the input model parameters, $p(D|\theta,M)$ is the likelihood function, $p(\theta|M)$ is the prior distribution of the input model parameters, and $p(D|M)$ is the evidence of the model class M in representing the experimental data D.
In this research a uniform distribution is adopted as the prior PDF distribution $p(\theta|M)$, and the likelihood function assuming stochastic independence measures is given by:

$$p(D|\theta,M) = p(f_1, f_2\,|\theta,M) = p(f_1\,|\theta,M)p(f_2\,|\theta,M) \quad (3)$$

$$p(f_i|\theta,M) = \frac{1}{\sqrt{2\pi}\sigma} exp\left[\frac{-1}{2}\left(\frac{f_i - g_i(\theta)}{\sigma}\right)^2\right], i = 1,2 \quad (4)$$

where $g_i(\theta)$ is the model output corresponding to frequency i with the model parameters θ, and σ is the standard deviation of the error between the experimental data and the output model.
To solve Eq. (2), the Metropolis-Hastings (M-H) algorithm (Metropolis et al, 1953; Hastings, 1970) is adopted for its simplicity and efficiency.

3. Results

3.1. *Operational Modal Analysis (OMA)*

A continuous vibration monitoring system was set up in the San Jerónimo Monastery (Granada) for the identification of the dynamic properties through OMA. These dynamic properties were identified by decoupling the effects that environmental conditions (mainly humidity and temperature) may have on them. The first and the second natural modes of vibration were obtained through OMA (from Artemis software package (ARTEMIS 2020)) as shown in Figure 2.

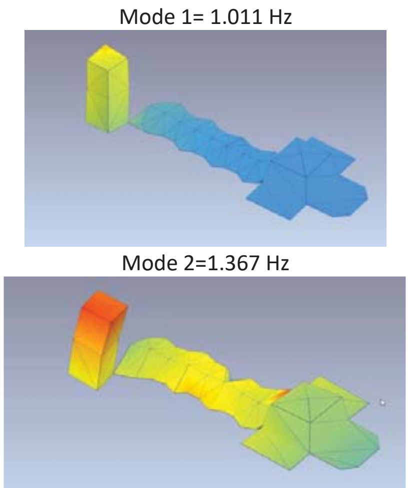


Figure 2. The first two natural frequencies and mode shapes.

3.2. *FEM of San Jerónimo Monastery*

As for the FE modelling of the CH building, the structure was modelled using shell-type elements with six degrees of freedom at each node. Both bending and membrane stiffness were considered. The mesh size was appropriately selected after a mesh convergence study. Just one element through thickness is considered in order to reduce the computational burden of the model. To include the soil stiffness in the FE analysis, 3-D uniaxial longitudinal spring-dampers with no mass were introduced at the level of the foundations. The FEM was parameterised considering the bulk modulus of the construction material (stone) and the reaction modulus of the soil as uncertain parameters. Figure 3 illustrates the structural FE model of the CH building.

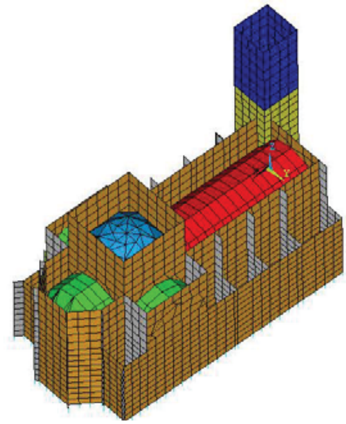
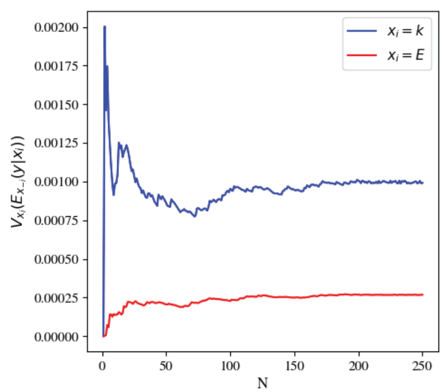


Figure 3. FE model and mesh details of San Jerónimo Monastery (Granada, Spain).

3.3 *Parameter sensitivity by Global Sensitivity Analysis (GSA)*

The variation of the model output as a consequence of variations in the bulk modulus of the stone *E* and the soil stiffness *k* parameters was analysed using the GSA method. Sobol' indices for those input parameters were obtained considering the first two numerical natural frequencies as model output. To compute Eq. (1), a double-loop and a single-loop Monte Carlo sampling were implemented to evaluate the numerator $V_{x_i}\left(E_{x_{-i}}(y|x_i)\right)$ and the denominator $V(y)$, respectively. In case of the first numerical frequency as output, n_1=n_2=250 samples were used for both the inner and outer loop in the numerator, whereas n_3=2000 samples were used in the denominator. Regarding to the second natural frequency as output, n_1=n_2=300 samples were adopted for both the inner and outer loop in the numerator, and n_3=1000 samples for the denominator. Figure 4 and Figure 5 show the convergence values of the Monte Carlo estimation of the numerator and denominator for the first and second numerical natural frequencies, respectively.



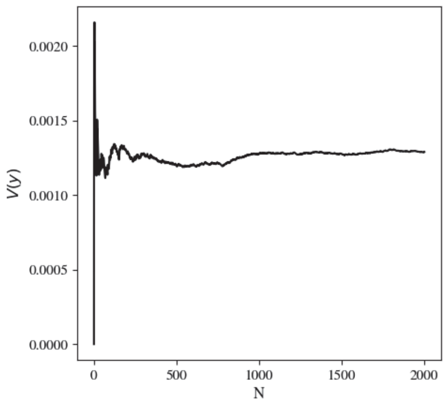


Figure 4 Convergence analysis for the first numerical natural frequency.

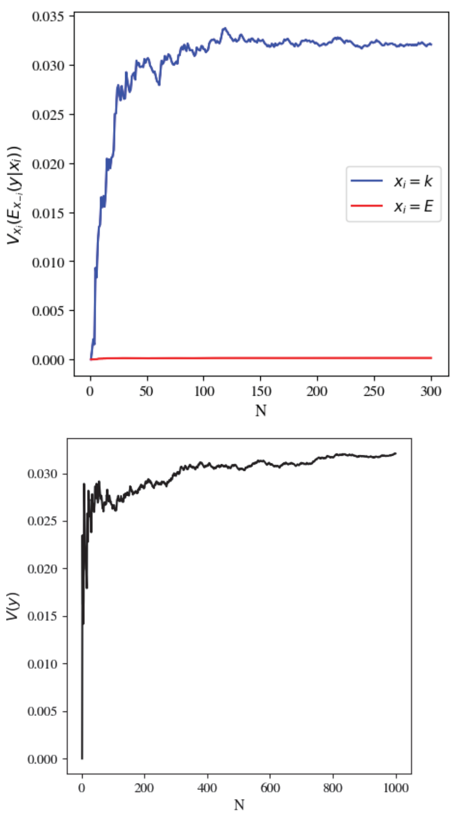


Figure 5. Convergence analysis for the second numerical natural frequency.

Table 1 shows the results of the Sobol' indices associated to the soil stiffness (S_k) and the bulk modulus of the stone (S_E).

It is observed that *k* has considerably higher effect than *E* in both natural frequencies.

Table 1. Sobol' indices for first two numerical frequencies for soil stiffness (Sk) and bulk modulus stone (S_E).

	1st frequency	2nd frequency
S_k	0.7881	0.9966
S_E	0.2078	0.0044
S_t	0.9959	1.0010

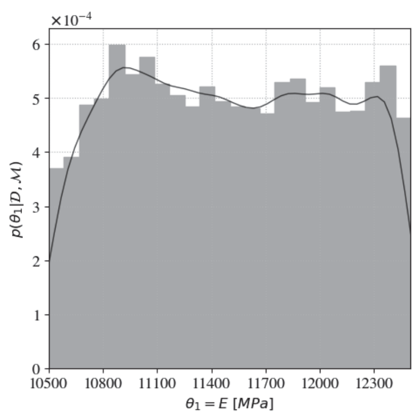
3.4 *Bayesian model parameters identification*

The Bayesian identification of model parameters is illustrated here. The prior information of the parameters is represented by uniform distributions as summarised in Table 2. Note that the standard deviation of the prediction error σ_e is assumed to be part of the set of uncertain parameters θ.

Table 2. Prior information of model parameters.

	Θ_1=E	Θ_2=k	$\Theta_3=\sigma_e$
Prior PDF $p(\theta\|M)$	U(10.5e³, 12.5e³)	U(5e⁵, 10e⁶)	U(0.01, 0.11)

Samples from the posterior PDFs of the model parameters were obtained using the M-H algorithm with 15,000 realisations. The posterior PDF results $p(\theta|D, M)$ of each individual parameter (θ_1=E, θ_2=k, $\theta_3=\sigma_e$) are represented in Figure 6. The PDF of the bulk modulus shows a relatively uniform behavior, which is aligned with the low sensitivity of this parameter. However, soil stiffness shows better identifiability as evident from in Figure 6, which is also consistent with the high sensitivity of this parameter.



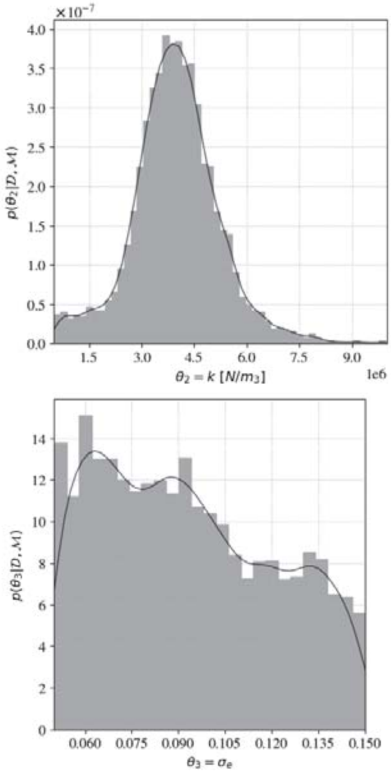


Figure 6 Posterior probability density functions of the model parameters.

Finally, the simulated first and second frequencies are obtained assuming the median of the inferred model parameters, namely E=11501.22 MPa and k=3945011.88 N/m^3. These simulated values are compared with the experimental ones in Table 3. The obtained results are still preliminary and part of a larger research, however they show the suitability and effectiveness of the developed method for model parameter identification from ambient vibration data.

Table 3. Experimental versus modelled frequencies for the median values of the model parameters.

	Experimental frequency(Hz)	Modelled frequency(Hz)	Error (%)
f_1	1.011	0.983	2.769
f_2	1.367	1.376	0.658

4. Conclusions

A Bayesian framework for FE model parameter identification in CH buildings based on modal data was presented in this paper. The methodology allows accounting for several sources of uncertainty such as the uncertainty coming from the variability of the measured data and the epistemic uncertainty of the FE model. The method was applied to San Jerónimo Monastery, a sixteenth century CH building of Granada (Spain). The building was instrumented and its experimental natural frequencies were obtained from Operational Modal Analysis (OMA). Basic mechanical parameters such as the bulk modulus of the stone and the soil stiffness were selected as uncertain parameters and a Global Sensitivity Analysis was applied, showing the higher relative importance of the soil stiffness for the first two natural frequencies of the building. Finally, the obtained modelled frequencies showed the suitability and effectiveness of the Bayesian methodology in identifying the plausible values of the uncertain parameters with quantified uncertainty.

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